# Exercise 84

(a) Find an equation of the tangent to the curve $y = e^x$ that is parallel to the line $x - 4y = 1$.

(b) Find an equation of the tangent to the curve $y = e^x$ that passes through the origin.

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**Solution**

## Part (a)

Solve the equation of the line for $y$.

$$x - 4y = 1 \quad \rightarrow \quad -4y = -x + 1 \quad \rightarrow \quad y = \frac{1}{4}x - \frac{1}{4}$$

It has a slope of $1/4$. Now take the derivative of the curve and set $y' = 1/4$.

$$y' = \frac{d}{dx}(e^x)$$

$$\frac{1}{4} = e^x$$

Solve for $x$.

$$\ln\frac{1}{4} = \ln e^x$$

$$\ln\frac{1}{4} = x$$

Now plug this value of $x$ into the equation of the curve to determine the corresponding $y$-value.

$$y = e^{\ln(1/4)} = \frac{1}{4}$$

Consequently, the point that the tangent line touches the curve at is

$$\left(\ln\frac{1}{4}, \frac{1}{4}\right).$$

Use the point-slope formula to obtain the equation of the tangent line with slope $1/4$.

$$y - \frac{1}{4} = \frac{1}{4}\left(x - \ln\frac{1}{4}\right)$$

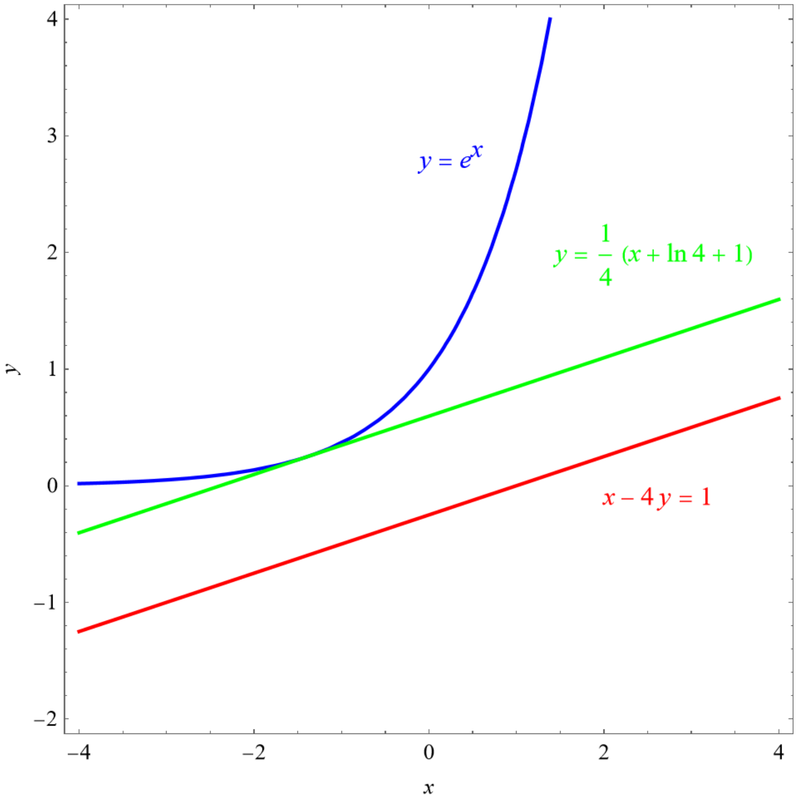
$$y - \frac{1}{4} = \frac{1}{4}x - \frac{1}{4}\ln\frac{1}{4}$$

$$y = \frac{1}{4}x - \frac{1}{4}\ln\frac{1}{4} + \frac{1}{4}$$

$$y = \frac{1}{4}\left(x - \ln\frac{1}{4} + 1\right)$$

$$y = \frac{1}{4}(x + \ln 4 + 1)$$

The graph below illustrates the curve and its tangent line with slope 1/4.




## Part (b)

Differentiate the equation of the curve.

$$y' = \frac{d}{dx}(e^x)$$

$$= e^x$$

Set $x = 0$ to determine the slope of the tangent line.

$$y'(0) = e^0 = 1$$

The slope is 1. Find the $y$-coordinate corresponding to $x = 0$.

$$y = e^0 = 1$$

Consequently, the point that the tangent line touches the curve at is

$$(0, 1).$$

Use the point-slope formula to obtain the equation of the tangent line.

$$y - 1 = 1(x - 0)$$

$$y - 1 = x$$

$$y = x + 1$$

The graph below illustrates the curve and its tangent line at $x = 0$.